Title: Aviation – Fuel requirements for aircraft

Topics: percentage, modelling, speed, distance, time, mass, density	**Time**: 90 minutes	**Age**: 13-14

Differentiation:

Higher level: air resistance can be taken into consideration

Lower level: Hints on worksheets can be used, parts of solution sheets can be provided

Guidelines, ICT support etc.:

An applet for calculating the amount of fuel is available

Optionally, the students may divide the group work amongst them, where one part concentrates on the mathematics aspects and the other part on the science aspects of the work sheets

Equipment needed for this activity:

Work sheets

Internet access

Required knowledge:

Basic operations with numbers

Concepts and correlations of speed, distance, and time

Concept of mass and density

Health and Safety:

Learning outcomes for this activity:

Students should be able to do calculations with given data and instructions

Students should be able to use the input of this lesson to make reasonable estimates about fuel consumption of aircraft, also in comparison with other means of transport

Students should be able to explain different ways of saving fuel (change altitude, reduce speed), as well the limitations of these approaches

Lesson description

Starter Activity

Students get some data about a particular flight (distance, average speed, see work sheet 1) and are asked to estimate the amount of fuel that would be required for this flight. Several estimates are written on the board.

After this, students raise their hands if they think each estimate is a) too low or b) to high. Number of students answering a) or b) is written next to the estimate.

Main Activity

Students form groups of 3. Each group receives work sheet 2 with specific data for this flight, as well as explanations how the actual amount of fuel for a flight is calculated. Students are then asked to perform these calculations with the given data of this flight. At the end of this activity, each group is to hand in their results to the teacher. Results (depending on rounding etc., it is quite likely that more than one "correct" result is achieved by different groups) are written on the board and are compared with the estimates from the starter activity.

Plenary Activity

Brainstorming session about what possibilities students see to save fuel.

Students are then to choose one of two work sheets for homework. Work sheet 3a includes some more calculations that are important for pilots, including fuel-saving options. Work sheet 3b asks for comparisons of aircraft fuel usage with fuel consumption of other means of transportation. The results are then to be discussed in the next lesson.

Aviation – Fuel requirements for aircraft

Work sheet 1 – starter activity

Aircraft require fuel, just like any other means of transportation (even if the “fuel” is sometimes provided by human muscular power). But how much fuel does an aircraft actually consume? And how does this compare with e.g. driving a car or sailing in a ship? Let’s find out!

Task: A Boeing 737-800 (which is a typical medium-range aircraft used by many airlines) needs to fly from Vienna to Dubai. The distance between these two cities is 2450 NM (nautical miles), which corresponds to 4537 km. The average speed during this flight is about 445 knots (nautical miles per hour).

Estimate how much fuel is needed for this flight!

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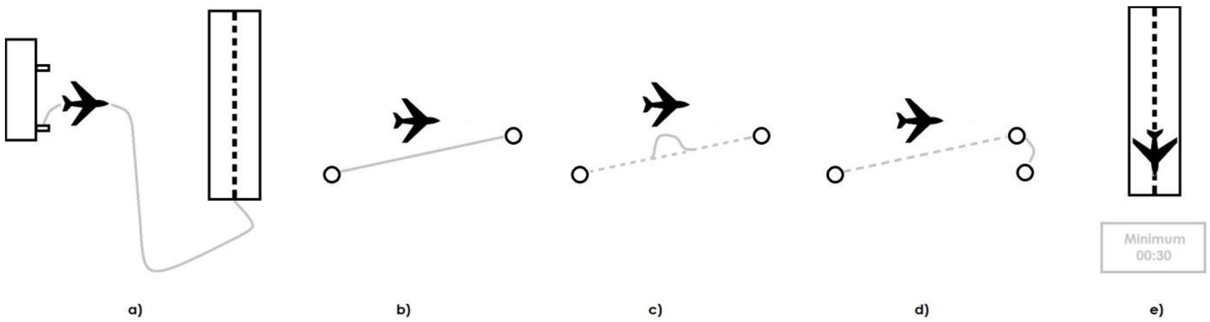
Work sheet 2 – main activity



The necessary amount of fuel for a commercial airliner consists of several parts that are calculated before each flight. The minimum amount of fuel comprises:

a) Fuel for taxiing from the parking position to the runway (flat-rate: 200 kg)

b) Fuel for the flight from departure airport to destination airport

c) 5% of b) as spare to compensate for changing winds, lower altitude, an eventual detour because of lightning etc.

d) Fuel for the flight from the destination airport to the alternate airport (if there is e.g. bad weather or a blocked runway at the destination airport), done with a standard speed of 380 kts (knots … nautical miles per hour)

e) Minimum remaining fuel (after landing there has to be fuel left for 30 minutes of flying)

Task 1: A Boeing 737-800 flies from Vienna to Dubai. As we already know, the distance between these two cities is 2450 NM (nautical miles) = 4537 km. The alternate airport for Dubai is Muscat, at a distance of 160 NM from Dubai. The aircraft has two engines; each one is using 1200 kg of fuel per hour. Calculate the minimum amount of fuel required for this trip! Hint: Calculate the fuel for the flight in b), then calculate and add the additional fuel for c), d), and e), then add the taxiing fuel from a) to it.

Task 2: A typical take-off mass of a Boeing 737-800 is 50000 kg. How much work is needed to accelerate the plane uniformly from standstill to take-off speed (approx. 160 kts) in 20 seconds, when you ignore air resistance?

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Work sheet 3a – homework activity (option a)

Now we know how much fuel an airplane needs to fly from Vienna to Dubai. But how long does it actually take to refuel a plane? Is there any way you can save fuel while flying?

Tasks:

a) At large airports fuel is pumped into the planes using underground pipes. However, airports measure fuel by volume, not by mass. How many liters of fuel have to be filled into the tanks if the density of jet fuel is 0.79 kg/l?

b) How long does the fuelling process take if the flow rate is 14 l/sec and there were 3.2 tons of fuel left in the tanks?

c) The pilots think of flying lower than planned, because the tailwind component is stronger by 25 kts if they fly 4000 feet lower. Is this option saving fuel, considering the fact that for each 1000 feet lower altitude 1% more fuel is required? How much fuel can be saved, if any? (Hint: Add the tailwind speed to the plane speed, and then repeat the calculations on work sheet 2, taking into consideration that the engines now need 4% more fuel).

d) The pilots think of reducing the airspeed to save fuel. How much (in %) can they reduce the airspeed if the maximum permitted flying time (to avoid delays) is 5 hours 46 minutes? How much fuel can they save by this, considering that 1% reduction in airspeed means 1% less fuel consumption? (Hint: Use the maximum flying time of 5 hours 46 minutes, and then calculate the fuel requirements as in work sheet 2).

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Work sheet 3b – homework activity (option b)

Now we know how much fuel an airplane needs to fly the 4537 km from Vienna to Dubai. It sounds like a lot of fuel, but how would this compare to driving a car the same distance? And a cruise ship?

Tasks):

a) An average car needs about 7 l (5.5 kg) of fuel to drive 100 km. How much fuel would the car need to go all the way from Vienna to Dubai in a straight line (4537 km)?

b) The most direct road connection between Vienna and Dubai is 5424 km long. How much fuel would the car in a) need to cover this distance?

c) The results from a) and b) hopefully reveal that it takes less fuel to drive a car from Vienna to Dubai than to fly a Boeing 737-800. But the car can take an average of 4 persons, while the plane can carry 189 passengers. How much fuel is needed per passenger for the car ride, how much for the plane ride?

d) Use the internet to find out the fuel consumption of an average cruise ship. Then calculate the amount of fuel that is needed to go via cruise ship from Genoa (the closest cruise ship harbor to Vienna) to Dubai, which (by ship) is 7680 km away. Calculate the fuel per passenger and compare it with the results from b) and c).

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Solution sheet

- Work sheet 2, task 1

  First calculate b), the amount of fuel to fly from departure airport to destination airport. For this, we calculate the flying time:

  time = distance/speed = 2450/445 ≈ 5.5 h

  Then we calculate the fuel consumption of one engine in 5.5 h

  $1200 \cdot 5.5 = 6600$ kg for the duration of the flight.

  For two engines:

  $6600 \text{ kg} \cdot 2 = 13200$ kg

  c) Now we add 5% of this value as spare:

  $13200 \cdot (1+0.05) = 13860$ kg

  d) We have to add the fuel to fly from the destination airport (Dubai) to the alternate airport (Muscat) at a standard speed of 380 kts. For this, we have to calculate the flying time from Dubai to Muscat:

  time = distance/speed = 160/380 ≈ 0.42 h

  Fuel consumption of both engines for 0.42 h of flying:

  $1200 \cdot 0.42 \cdot 2 \approx 1000$ kg

  We add this to the result from c)

  13860 + 1000 = 14860 kg

  e) After landing, there have to be fuel for 30 minutes (0.5 h) left:

  $1200 \cdot 0.5 \cdot 2 = 1200$ kg

  Again we add this to the result of e)

  14860 + 1200 = 16060

  a) And finally, we add the fuel for taxiing from parking position to runway:

  16060 + 200 = **16250 kg**

- Work sheet 2, task 2

  First, we calculate the speed in standard units, i.e. m/s:

  $v = 160 \text{ kts} = 160 \text{ NM/h} = 160 \cdot 1.852 \text{ km/h} = 160 \cdot 1852 \text{ m/h} = 160 \cdot 1852/3600 \text{ m/s} \approx 82.3 \text{ m/s}$

  Now we can calculate the required amount of work:

  $W = m \cdot v^2/2 = 50000 \cdot 82.3^2/2 = 169332250 \text{ J} \approx$ **169 MJ**

- Work sheet 3a, task a

  Volume = mass/density = 16250/0.79 ≈ **20570 l**

- Work sheet 3a, task b

  First we calculate how much fuel has to be put into the tank:

  16250 – 3200 = 13050 kg

  With a flow rate of 14 l/s, this takes

  time = fuel/flow rate = 13050/14 ≈ 932 s ≈ **15 minutes**

- Work sheet 3a, task c

  We repeat the calculations of work sheet 2, with two changes: The tail wind speed is added to the planes' speed, and the engine's fuel consumption is increased by 4%. We start again by calculating b), the amount of fuel to fly from departure airport to destination airport. For this, we calculate the flying time:

  time = distance/speed = 2450/(445+25) ≈ 5.2 h

  Then we calculate the fuel consumption of one engine in 5.2 h

  1200·1.04·5.2 ≈ 6490 kg for the duration of the flight.

  For two engines:

  6490 kg·2 = 12980 kg

  c) Now we add 5% of this value as spare:

  12980·(1+0.05) ≈ 13630 kg

  d) We have to add the fuel to fly from the destination airport (Dubai) to the alternate airport (Muscat) at a standard speed of 380 kts. The tailwind component is not added here, so we can take the result from work sheet 2:

  time = distance/speed = 160/380 ≈ 0.42 h

  Fuel consumption of both engines for 0.42 h of flying:

  1200·1.04·0.42·2 ≈ 1050 kg

  We add this to the result from c)

  13630 + 1050 = 14680 kg

  e) After landing, there have to be fuel for 30 minutes (0.5 h) left, at the original altitude, so again we can take the result from work sheet 2:

  1200·0.5·2 = 1200 kg

  We add this to the result of e)

  14680 + 1200 = 15880

  a) And finally, we add the fuel for taxiing from parking position to runway:

  15880 + 200 = 16080 kg

  We compare this with the amount of fuel calculated at work sheet 2:

  16250 – 16080 = **170 kg**

  The pilots save 170 kg of fuel, i.e. approx. 1%.

- Work sheet 3a, task d

  The flying time of 5 hours 46 minutes = 5 + 46/60 hours = 5.77 h
  We calculate the air speed:
  Speed = distance/time = 2450/445 ≈ 424.6 kts
  How much less is this compared to the original air speed of 445 kts
  (424.6-445)/445 ≈ -4.6 %
  The plane is 4.6% slower, i.e. the engines need 4.6% less fuel.
  Now we calculate the fuel consumption of one engine in 5.77 h, taking into consideration that the engines now need 4.6% less fuel:
  1200·5.77·(1-0.046) ≈ 6600 kg for the duration of the flight.
  For two engines:
  66000 kg·2 = 13200 kg
  We can stop the calculation here, as we can already see the planes uses the same amount of fuel as on work sheet 2. **No fuel can be saved** by flying slower in this case.

- Work sheet 3b, task a

  4537/100·5.5 = **249.5 kg**

- Work sheet 3b, task b

  5424/100·5.5 = **298.3 kg**

- Work sheet 3b, task c

  Fuel usage of **plane** per passenger:
  16250/189 ≈ **86 kg/passenger**
  Fuel usage of **car** per passenger:
  298.3/4 ≈ **75 kg/passenger**