Title: GPS – Find your way

Topics: intersection of spheres, coordinate systems, distance, speed and time, signal transmission	**Time**: 90 minutes	**Age**: 16+

Differentiation:

Higher level: Error correction in GPS receivers can be discussed

Lower level: Intersection of spheres can be done with GeoGebra or similar software

Guidelines, ICT support etc.:

It is strongly suggested for students to collect their own data with their own devices. For this, they should be advised to collect GPS data before the start of lesson 1 and bring it with them. Should this not be possible, either the teacher can provide the data, or trip data from (e.g.) hiking or biking sites on the internet can be downloaded.

The theoretical introduction (work sheet "basics") may need more input by the teacher, depending on students' pre-knowledge.

Equipment needed for this activity:

Work sheet

Internet access

GPS device or GPS app on smartphone

Required knowledge:

3D-geometry

Transmission time for signals

Health and Safety:

Work sheets contain tasks for outdoor activities. Safety issues to be discussed with students beforehand (e.g. avoid using handheld GPS device while biking)

Learning outcomes for this activity:

Students should be able to understand the underlying principles of GPS

Students should be able to perform the calculations and tasks on the work sheets and present the facts in a suitable way to their peers

Students should be able to use a GPS device in everyday life situations, and know about the limits of the system

Lesson description

Starter Activity

At the beginning of lesson 1, the topic can be introduced either by a short theoretical exposition period about GPS technology, or by showing headlines of GPS related “accidents” (car stuck on stairways etc.).

Main Activity

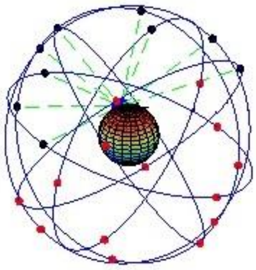
Students form three (in larger classes six) teams. Teams choose “basics”, “fit”, or “fun” as a topic and receive the corresponding work sheets. The teams now have time to read, understand, and summarize the work sheet content, and start to prepare a poster and a 5 minute presentation for their classmates. This concludes lesson 1.

Plenary Activity

In lesson 2, each team will step out and present the results of the group work to the entire class. Each 5 minute presentation is followed by a 5 minute question-and-answer session (mainly conducted by the students; the teacher(s) should only involve him-/herself if either answers are not correct or important facts are not covered). At the end of this session, students should have the most important facts of the topics covered.

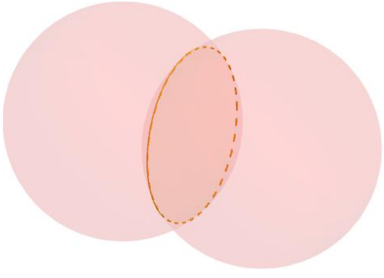
GPS – find your way

Work sheet 1 – basics (How GPS works)

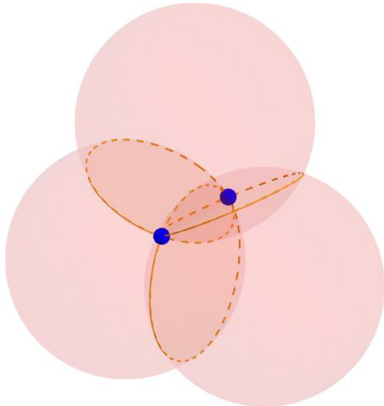


GPS (Global Positioning System) is based on three-dimensional geometry, using a number of satellites that are in earth orbit. These satellites are continually sending signals that can be received and interpreted by a GPS receiver (sometimes also called a GPS device, or simply – but less accurately – a GPS). The signals contain the satellites position, the time when the signal was sent, and additional information about satellite health and the other satellites. With the help of the signals of (usually) at least four satellites, it is possible to calculate the position of the GPS receiver. Not counting error corrections (which make for a fairly complicated calculation), the calculation of position works like this: The receiver computes the difference between the time t_s the signal was sent by the satellite and the time t_r the signal was received by the device. As the signal travels with the speed of light c, this allows the computation of the distance d to the satellite as $d = c \cdot (t_r - t_s)$. Now we know that we are (or actually the GPS device is) at a distance of d to the first satellite. We also know this satellites' position, so we only need to think "what is the set of points with a given distance to a fixed point?" In the plane, the answer would be "a circle", but as we are in (three-dimensional) space, the answer is "a sphere" (to be more exact: "the surface of a sphere"). So, with only one satellite signal, we would only know that we are somewhere on the surface of this (virtual) sphere.

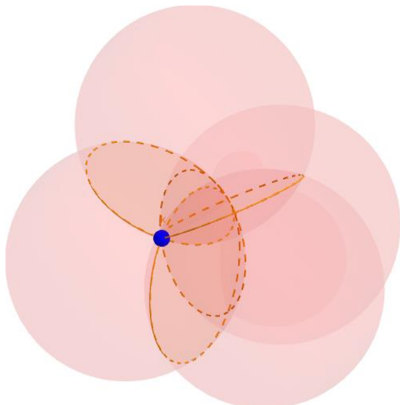
With the signal of a second satellite, we can construct a second sphere, and now we know that we are on the surface of both of these spheres, i.e. on the intersection of two sphere surfaces. The intersection of two sphere surfaces is a circle.



A third satellite provides us with yet another sphere, and the intersection of these three surfaces is a set of two points.



If these two points are far away from each other, this might be enough, as we usually have other information to decide which one of these two points is our actual position (e.g. usually you would know whether you are somewhere in Austria or somewhere near the South Pole). But then, the two points may be fairly close together, and it would be nice to know whether you are near your hotel or have another 20 km to hike. So for all practical purposes we need the signal from a fourth satellite, which results in exactly one point (again, we ignore possible errors and their correction and remain within the description of an ideal case).



Task 1: The speed of the GPS signal is approx. 300,000 km/s. The satellites orbit at an altitude of 20,200 km. How long does the signal take to reach earth?

Task 2: Each satellite has an orbital period of 11 h 58 min (½ sidereal day). What's the speed of such a satellite with respect to Earth surface?

GPS – find your way

Work sheet 2 – fit (Collecting and analyzing data with GPS)

Data can be easily obtained by switching a GPS device to record the data. As this is done differently with different GPS models, we will not show this in detail here. Just take your GPS device (or your Smartphone with a GPS App) and read the manual, or try it out. Also, find out how to transfer the data from your GPS device or Smartphone onto your computer.

Task 1: If you have a GPS device or GPS-enabled Smartphone, use it and collect the data of a hiking, biking, or walking trip! Alternatively, download some GPS trip data from the internet.

Task 2: In class, transfer the data to your computer and use spreadsheet software to find out the following:

a) What was the length of the trip?
b) How long did the trip take?
c) What was your average speed?

Task 3: Transfer the data to Google Earth and create a satellite image with the track of your trip in it!

Remark: Details on how to transfer and analyze GPS data with Excel can also be found here:

http://www.dm.unipi.it/~georgiev/club/progects/DYNAMAT/PUBLIC/D9_EBook/PDF_English/AT_EN_1_B_Aviation.pdf

GPS – find your way

Work sheet 3 – fun (GeoCaching: Find something with GPS)

A lot of people use GPS devices to find out their own position, or to find a route between two points. But you can do something else with your GPS device. Not only does it allow you to find your way to a certain city or the next gas station, it also allows you to find so-called geocaches. Now, what is a geocache? A geocache is a container (including a logbook) of some sort (sizes range from very small ones the size of a screw head to very big ones the size of a bucket) that somebody hid, and then published its coordinates on a webpage. Your job is to use these coordinates and your GPS device or Smartphone to find the container, and then log this find both physically in the logbook and virtually on the webpage. Sounds easy, right? Well, often it is not as easy as you might think …

We just spoke about "publishing coordinates", but for that to be useful we have to first agree which coordinate system we use. In school, we are used to using mainly Cartesian coordinates. These are fine on a plane surface, but not very useful on the surface of the earth, which is essentially the surface of a sphere. In this case, we have to use a spherical coordinate system (basically the three-dimensional equivalent of the polar coordinate system). Each spherical coordinate system requires a fixed origin (from where the distance is measured) and two fixed planes (from where the angles of longitude and latitude are measured). In GPS navigation, people usually use the WGS84 system. Its fixed origin is the center of the Earth; the fixed planes are the plane through the equator and a plane through the "zero meridian" near Greenwich, UK. Each point therein can be described by two (if we just want to know the position on the earths' surface) or three (if we also want to know positions under water or in the air) coordinates: A latitude, a longitude, and (eventually) an altitude (usually, the altitude is used instead of the distance to the earth center). Both the latitude and the longitude are measurements of angle, which are usually given in degrees and (angular) minutes. The latitude is counted north or south of the equator, the longitude west or east of the zero meridian. A typical WGS84 position would look like this: N 48° 12.507, E 016° 22.331.

Task 1: Use Google Maps or Google Earth or a similar tool to find out where the position with the above coordinates actually is.

Task 2: Find out the WGS84 coordinates of your home and of your school.

Task 3: How many meters does your position change, if you vary the longitude by 1°?

Task 4 (optional): Now visit one of the several available GeoCaching websites (for instance http://www.opencaching.de or http://www.geocaching.com), choose a suitable GeoCache in your vicinity, go out with a GPS device and find it!