Title: Space flight – landing a Space Shuttle

Topics: exponentials, derivatives, temperature, speed, distance and time, air density, energy conversion	**Time**: 135 minutes	**Age**: 16+

Differentiation:

Higher level: Differential equation can be solved without CAS support, as shown on solution sheet 2a

Lower level: Hints or the full solution sheets can be provided, or the differential equation can be omitted

Guidelines, ICT support etc.:

Solution sheets are provided for work sheets 1 and 2. Depending on pre-knowledge and level of mathematics and science that should be covered, the teacher can decide to hand over the entire solution sheets to the students, provide hints from the solution sheets, or omit using the solution sheets in class altogether.

Solutions sheet 2 uses CAS to solve the differential equation, solution sheet 2a solves the equation "by hand"

Equipment needed for this activity:

Work sheet

Internet access

Optional: Model kit of a Space Shuttle

Required knowledge:

Concepts of types of energy

Functions and derivatives

Concept of "air friction"

Health and Safety:

Learning outcomes for this activity:

Students should be able to perform the calculations on the work sheets, eventually with CAS support and/or the help provided on the solution sheets

Students should be able to understand the problems and possible solutions of heat insulation

Students should be able to understand the physical principles of "air friction"

Students should be able to collaborate within and across teams to achieve results

Lesson description

Starter Activity

At the beginning of lesson 1, the teacher introduces the topic by showing video of lift-off and landing of a Space Shuttle. Though the Shuttle actually does not fly anymore, it still provides an interesting background for maths and science, and the principles of it are useful for any space vehicle. Then warm-up questions are asked: Do you think the Shuttle goes fastest at some point during lift-off, while it is in orbit, or at some point during landing (Answer: while it is in orbit)? Then why is it that the Shuttle has a “real hot” phase only during landing? How hot is “real hot”? And how long is this phase? Estimates are taken about the answers to the last two questions, and they are written on the board.

Main Activity

Students form 4 teams. Teams choose “temperature”, “time”, “insulation”, or “speed” as a topic and receive the corresponding work sheets. The teams now have time to read, understand, and summarize the work sheet content, and then prepare a poster and a 5 minute presentation for their classmates. This will take lessons 1 and 2 to finalize.

Plenary Activity

In lesson 3, each team will step out and present the results of the group work to the entire class. Each 5 minute presentation is followed by a 5 minute question-and-answer session (mainly conducted by the students; the teacher should only involve him-/herself if either answers are not correct or important facts are not covered). At the end of this session, students should have the most important facts of the topics covered.

The lesson can end in several ways after the presentation/question-answer session. Current space missions might be discussed, or history of Shuttle flights, or further questions can be answered by the teacher.

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Work sheet 1 – temperature

How hot does a Space Shuttle get during landing?

Before we can answer this question, we first have to see why a Space Shuttle is getting hot in the first place. The answer is popularly called air friction. Air molecules (mainly Nitrogen and Oxygen molecules) hit the surface of the solid, and a tiny fraction of the kinetic energy of the solid is converted into thermal energy (or kinetic energy of air molecules). With relatively slow speed this effect is also there, but too small to realize – if you move your hands through the air, you don't feel them. At the speed of an airplane, the effect is already measurable. At the speed of a Space Shuttle (about 20 times the speed of a commercial airliner), the effect becomes a major engineering problem.

Task: The "hottest" part of the landing is between the time of the de-orbit (i.e. the time when the landing process begins) at an altitude of 122 km and at a speed of $v_1 = 25{,}900\,\text{km/h}$, and the time the Shuttle exits from Radio blackout at an altitude of 55 km and a speed of $v_2 = 13{,}300\,\text{km/h}$. How much does the temperature of the Shuttle surface (the Shuttles' heat capacity is $c \approx 500\,\frac{\text{J}}{\text{kg K}}$) increase in this period? (Interesting fact: Because of the shock wave effect only 4 % of the converted energy heats up the Shuttle, the rest heats up the air and does not concern us here).

How can the Shuttle stand this temperature? Ask team "insulation"!

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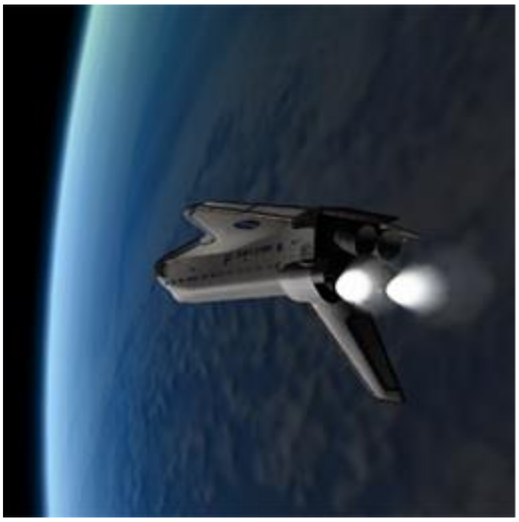
Work sheet 2 – time



How long is the "hot phase" during a Space Shuttle landing?

We already know that "air friction" is the main reason why a Space Shuttle gets hot while landing. It is also the main reason for the Shuttle getting slower. Air friction depends on several factors: Speed v (the main factor), air density ρ, (effective) surface area of the object A_{eff}, and the geometric form of the object (described by a form factor, or drag coefficient, c_d). The deceleration is given by

$$a = -\frac{1}{2m} \cdot \rho \cdot v^2 \cdot A_{eff} \cdot c_d$$

Mass, effective area, and form factor of the shuttle can be easily determined and remain fairly constant, but the air density depends on the altitude, weather etc. The air density in Earths' atmosphere at a certain altitude h (in m) is given by

$$\rho_h = \rho_0 \cdot e^{-h \cdot 0.00011856} \text{, where } \rho_0 = 1.2250 \frac{\text{kg}}{\text{m}^3} \text{ is the air density at sea level}$$

Task: The "hottest" part of the landing is between the time of the de-orbit (i.e. the time when the landing process begins) at an altitude of 122 km and at a speed of $v_1 = 25{,}900$ km/h, and the time the Shuttle exits from Radio blackout at an altitude of 55 km and a speed of $v_2 = 13{,}300$ km/h. The wing area of a space shuttle is 250 m^2, it is coming in with its nose tilted upwards at about 40°, its mass at landing is approximately 100 t, and the drag coefficient is about 0.078. (Remark: For reasonably simple calculations, consider the air density being a constant value $\rho_{55\text{km}}$).

Now we know the speed at time of re-entry and the speed at time of radio blackouts' end, but how fast is the maximum speed? Ask team "speed"!

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Work sheet 3 – insulation

How can the Space Shuttle stand the heat?

The temperature of the hottest parts of the Space Shuttle during landing is > 1,500 °C. This means that using regular airplane materials for the shuttle surface would not be enough to protect it from the re-entry heat (steel melts at 1530 °C, aluminum melts at 660 °C, and the polycarbonate that airplane windows is made of melts already at 155 °C). That's why the surface of the shuttle (particularly the parts getting hottest, i.e. the nose cap and the wing leading edges, and to a lesser extend the underside of the main body and of the wings) is covered with a Thermal Protection System – reinforced carbon-carbon at critical places, insulation tiles made of silica ceramics (which has a high melting point and sheds heat very quickly, see the right picture above), and flexible insulation blankets at "cooler" places.

Task 1: Find out which material is used in which places on the Shuttle! For which temperature range is each material used?

Task 2: What is the major difference between the Shuttles' Thermal Protection System and the system that was used on earlier spacecraft (e.g. the Apollo capsule)? Why do you think the system was changed?

How hot does the shuttle get exactly? Ask team "temperature"!

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Work sheet 4 – speed

How fast does the Space Shuttle go?

As you will see, the Space Shuttle definitely does not have a constant speed, so the question "how fast does it go?" immediately needs to be clarified by asking "at what point in time?" as well as "in relation to what?" – after all, speed is relative! Let's for the rest of this lesson assume that we talk about speed in relation to Earth.

Task 1: From the moment of take-off, for about 26 seconds, the Shuttle has an average acceleration (mainly provided by the Solid Rocket Boosters [SRB], which are the two white cylindrical rockets at the side of the Shuttle, and to a lesser degree, by the Main Engines) of about 16.6 m/s^2. How fast is the Shuttle when it "clears the tower", i.e. when the lowest part of the Shuttle passes the highest part of the launch pad (distance about 105.8 m)?

Task 2: 124 seconds into the flight, the SRBs are burned out and separated from the Shuttle, which from then on runs on the Main Engines alone. At this time, the speed is 5,650 km/h. Calculate the average acceleration of the Shuttle from take-off to SRB separation.

Task 3: At the end of the launch phase, 8 minutes 30 seconds after take-off, the Main Engines are shut down (MECO = Main Engine Cut-Off) and the shuttle reaches its (almost) final and maximum speed of 29,000 km/h (Interesting fact: Some speed increase might occur due to firing the thrusters, but this is minimal). What is the average acceleration of the Shuttle from SRB separation to MECO? And the average acceleration during the whole launch?

That was the launch! And how fast is the Shuttle at the beginning and end of the "hot phase" during landing? How long is this phase? Ask team "time"!

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Solution sheet 1 – temperature

$$E_{kin} = \frac{m \cdot v^2}{2} \text{ … kinetic energy}$$

$$E_{th} = m \cdot c \cdot T \text{ … thermal energy}$$

Energy conversion:

Difference of kinetic energy = difference of thermal energy

Now we take into consideration that only 4% of the converted energy heat up the shuttle:

Difference of thermal energy = 4% of difference of kinetic energy

$$\Delta E_{th} = 0.04 \cdot \Delta E_{kin}$$

$$\not{m} \cdot c \cdot \Delta T = 0.04 \cdot \left(\frac{\not{m} \cdot v_1^{\ 2}}{2} - \frac{\not{m} \cdot v_2^{\ 2}}{2} \right) = 0.04 \cdot \frac{\not{m}}{2} \cdot \left(v_1^{\ 2} - v_2^{\ 2} \right)$$

$$\Delta T = \frac{0.04 \cdot \frac{v_1^{\ 2} - v_2^{\ 2}}{2}}{c}$$

As we are doing the calculations in the metric system, all units (here: particularly the speed) have to be converted into standard units. For speed, this is m/s. The factor of conversion between km/h and m/s is 3.6, i.e. 1 m/s = 3.6 km/h. This leads to $v_1 = 7{,}194$ m/s and $v_2 = 3{,}700$ m/s. Now we can calculate the temperature difference:

$$\Delta T = \frac{0.04 \cdot \frac{v_1^{\ 2} - v_2^{\ 2}}{2}}{c} = \frac{0.04 \cdot \frac{7{,}194^2 - 3{,}700^2}{2}}{500} = 1{,}522$$

The temperature difference is 1,522 K. As degrees Celsius and Kelvin have the same relative scale units, we can also write: The temperature difference is 1,522 °C.

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Solution sheet 2 – time

We start with find the function $v(t)$, giving the relation between speed v and time t. To find this relation we remind ourselves that deceleration is the change of speed with time, i.e.

$$a = \frac{\mathrm{d}v}{\mathrm{d}t}$$

With the above equation for deceleration by air friction we get

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{1}{2m} \cdot \rho \cdot v^2 \cdot A_{eff} \cdot c_d$$

This is a differential equation, which can e.g. be solved using a Computer Algebra System. The solution is

$$v(t) = \frac{1}{\frac{1}{2m} \cdot \rho \cdot A_{eff} \cdot c_d \cdot t - 0.00014}$$

Now we only have to calculate the remaining variables: the air density ρ, and the effective surface area of the object A_{eff} (mass and drag coefficient, as well as the speed at the time the shuttle leaves the “hot phase”, are known). As for the air density we made the assumption that this is a constant value $\rho_{55\mathrm{km}}$:

$$\rho_{55\mathrm{km}} = \rho_{55{,}000\mathrm{m}} = \rho_0 \cdot e^{-55{,}000 \cdot 0.00011856} = 0.0018\,\frac{\mathrm{kg}}{\mathrm{m}^3}$$

As for the effective area one would think this might be the same as the wing area (250 m^2), but the Shuttle is coming in at an angle of 40° the actual angle varies due to several flight maneuvers performed by the onboard computer, but for most of the re-entry it is indeed 40°), i.e. the effective area must be reduced by the factor sin 40° (if you look at a sheet of paper from a 90° angle you see the full area, from another angle the area seems to be smaller):

$$A_{eff} = 250\,\mathrm{m}^2 \cdot \sin 40° = 160\,\mathrm{m}^2$$

Finally we enter all the values into the equation for speed and calculate the time t:

$$t = \frac{1 + v(t) \cdot 0.00014}{v(t) \cdot \frac{1}{2m} \cdot \rho \cdot A_{eff} \cdot c_d} = 3{,}653\,\mathrm{s} \approx 60\,\mathrm{min}$$

The shuttle takes about 60 minutes from the de-orbit burn to the end of radio blackout.

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Solution sheet 2a – time

We start with find the function *v*(*t*), giving the relation between speed *v* and time *t*. We remind ourselves that deceleration is the change of speed with time, i.e. $a = \frac{\mathrm{d}v}{\mathrm{d}t}$

With the above equation for deceleration by air friction we get

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{1}{2m} \cdot \rho \cdot v^2 \cdot A_{eff} \cdot c_d$$

This is a differential equation of the form

$$\dot{v} + k \cdot v^2 = 0 \text{, with } k = \frac{1}{2m} \cdot \rho \cdot A_{eff} \cdot c_d$$

The solution can easily be found, e.g. by using separation of variables

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -k \cdot v^2 \Rightarrow \frac{1}{v^2} \cdot \mathrm{d}v = -k \cdot \mathrm{d}t \overset{\int}{\Rightarrow} -\frac{1}{v} = -k \cdot t + c \text{, i.e. } v(t) = \frac{1}{k \cdot t - c}$$

As we have the border condition *v*(0) = v_1 = 25,900 km = 7,194 m/s, we get *c* = –0.00014, and the function *v*(*t*) for speed is

$$v(t) = \frac{1}{\frac{1}{2m} \cdot \rho \cdot A_{eff} \cdot c_d \cdot t - 0.00014}$$

Now we only have to calculate the remaining variables: the air density ρ, and the effective surface area of the object A_{eff} (the other values are known). As for the air density we made the assumption that this is a constant value $\rho_{55\mathrm{km}}$:

$$\rho_{55\mathrm{km}} = \rho_{55{,}000\mathrm{m}} = \rho_0 \cdot e^{-55{,}000 \cdot 0.00011856} = 0.0018 \frac{\mathrm{kg}}{\mathrm{m}^3}$$

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$$A_{eff} = 250\,\mathrm{m}^2 \cdot \sin 40° = 160\,\mathrm{m}^2$$

Finally we enter all the values into the equation for speed and calculate the time *t*:

$$t = \frac{1 + v(t) \cdot 0.00014}{v(t) \cdot \frac{1}{2m} \cdot \rho \cdot A_{eff} \cdot c_d} = 3{,}653\,\mathrm{s} \approx 60\,\mathrm{min}$$

The shuttle takes about 60 minutes from the de-orbit burn to the end of radio blackout.